Probing inflation via the gravitational wave background

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in collaboration with

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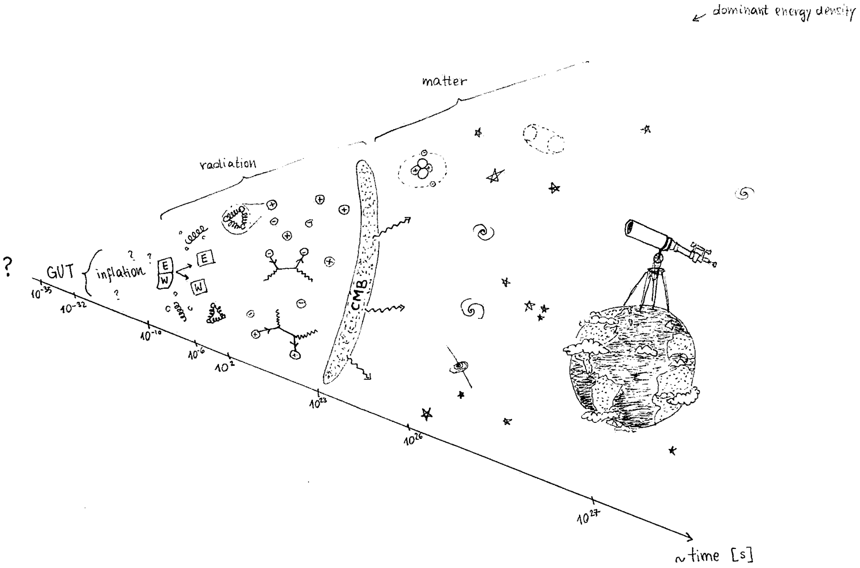
12.06.2023


dominant energy density
matter
radiation
?
GUT
inflation
E
W
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W
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10^{-35}
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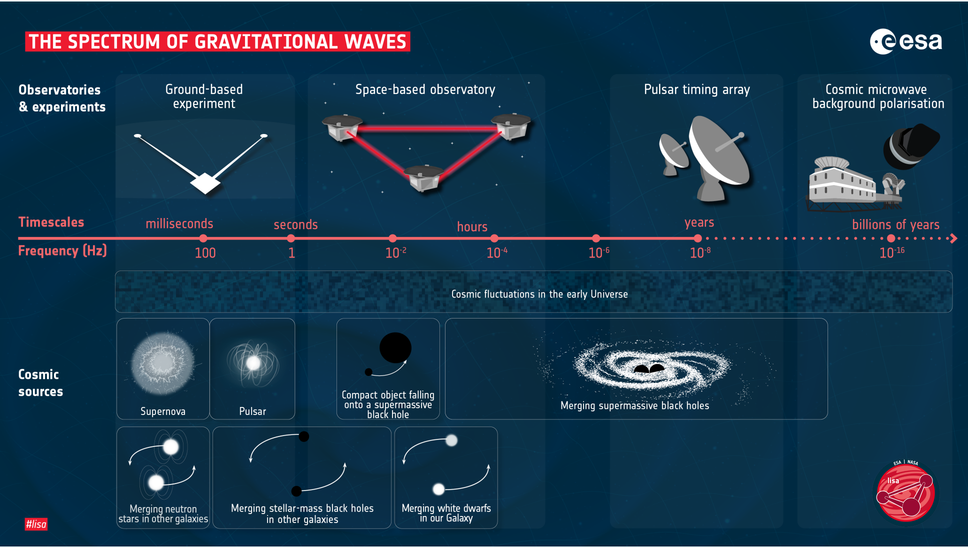
~time [s]



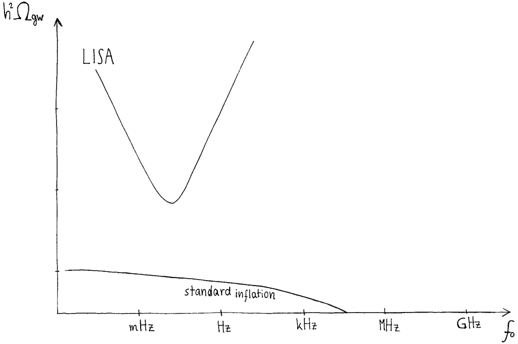
www.esa.int/ESA Multimedia/Images/2021/09/The spectrum of gravitational waves

Laser Interferometer Space Antenna

* frequency window $f_0 \sim (10^{-4} - 10^{-1})$ Hz

* noise → primordial background

* probe **inflation:**



Big-Bang cosmology: homogeneous + isotropic universe

$$ds^2_{\mathtt{FLRW}} = -\mathrm{d}t^2 + a(t)^2\,\mathrm{d}\mathbf{x}^2_\kappa\,, \quad H \equiv \frac{\dot a}{a} > 0$$

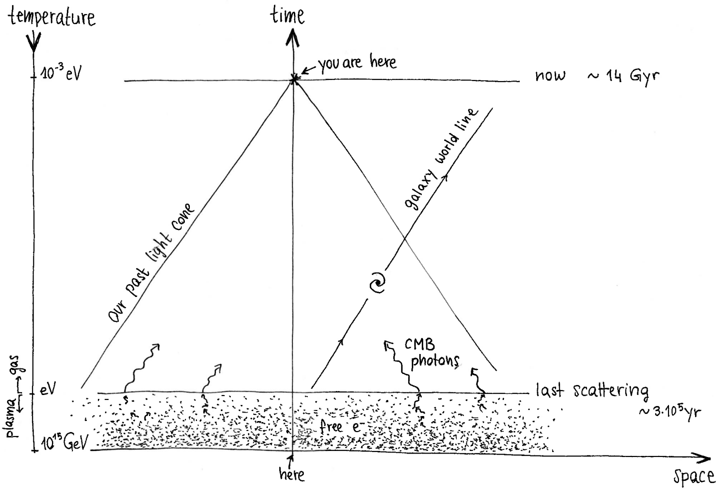
$$T^\mu{}_\nu = \begin{pmatrix} e & \\ & -p\,\delta_{ij} \end{pmatrix}$$

$$\curvearrowright \quad G^\mu{}_\nu = 8\pi G\, T^\mu{}_\nu \quad \Rightarrow \quad H^2 = \frac{8\pi}{3m^2_{\mathtt{pl}}} e$$

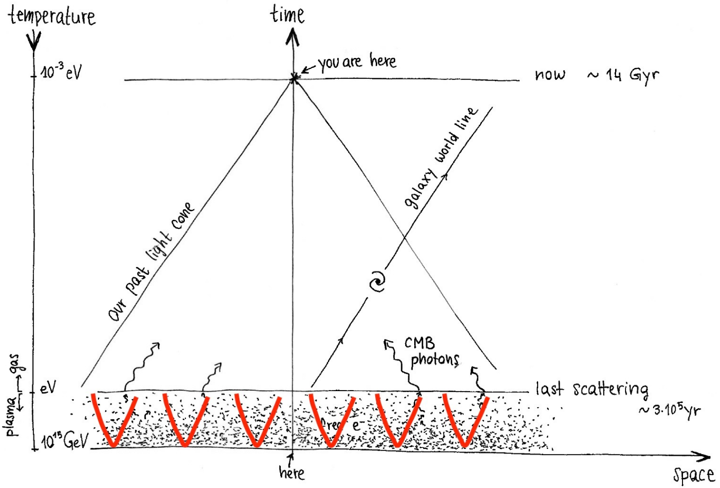
$$\frac{\ddot a}{a} = -\frac{4\pi}{3m^2_{\mathtt{pl}}}(e+3p) < 0$$

matter, radiation: $p \geq 0$

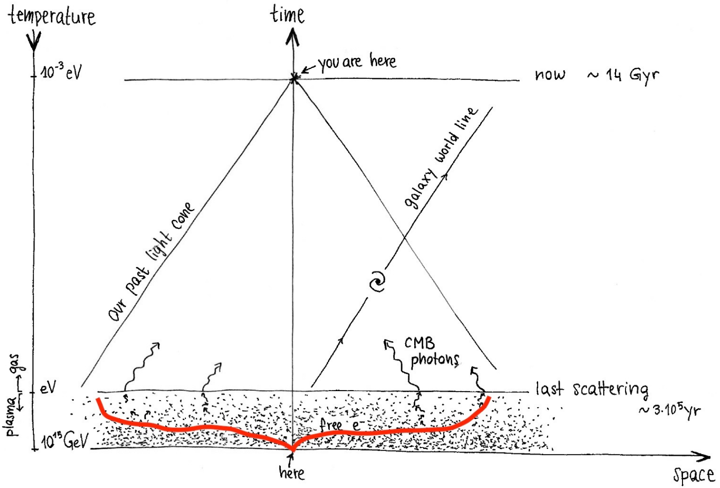
flatness + horizon problems



flatness + horizon problems



inflation: early accelerated expansion



inflation: early accelerated expansion

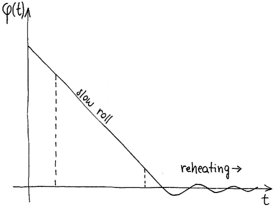
$G^{\mu}{}_{\nu} = 8\pi G \, T^{\mu}{}_{\nu}$

$$H^2 = \frac{8\pi}{3m_{\mathrm{pl}}^2} e$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{\mathrm{pl}}^2}(e + 3p) > 0 \quad \Leftrightarrow \quad p < 0$$

parametrize with scalar field

$$\mathcal{L} = \frac{1}{2}\partial^{\mu}\varphi \, \partial_{\mu}\varphi - V(\varphi)$$



$$p = \frac{\dot{\varphi}^2}{2} - V(\varphi) \approx -V(\varphi) \,,$$

$$e = \frac{\dot{\varphi}^2}{2} + V(\varphi) \approx V(\varphi)$$

inflation: early accelerated expansion

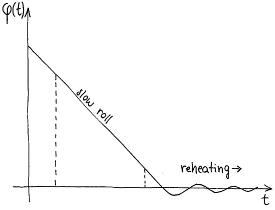
$G^\mu{}_\nu = 8\pi G\, T^\mu{}_\nu$

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2} e \quad \Rightarrow \quad H \equiv \frac{\dot a}{a} \approx \text{const.}$$

$$\frac{\ddot a}{a} = -\frac{4\pi}{3m_{\rm pl}^2}(e+3p) > 0 \quad \Leftrightarrow \quad p < 0$$

parametrize with scalar field

$$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - V(\varphi)$$



$$p = \frac{\dot\varphi^2}{2} - V(\varphi) \approx -V(\varphi)\,,$$

$$e = \frac{\dot\varphi^2}{2} + V(\varphi) \approx V(\varphi) \approx \text{const.}$$

GWs $\leftrightarrow$ space-time perturbations

$$\underbrace{\bar{g}_{\mu\nu}}_{\text{flat FLRW}} + \delta g_{\mu\nu} + \mathcal{O}(\delta^2) = \begin{pmatrix} -1 & \\ & a^2(\delta_{ij} + h^t_{ij}) \end{pmatrix}$$

$$\Rightarrow \underbrace{\delta G^{\mu}{}_{\nu}}_{\nearrow\, h^t_{ij}} = 8\pi G \underbrace{\delta T^{\mu}{}_{\nu}}_{\nwarrow\, \delta T^t_{ij}}$$

$\Rightarrow$ tensor perturbations satisfy

$$\boxed{\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right) h^t_{ij} = 16\pi G\, \delta T^t_{ij}}$$

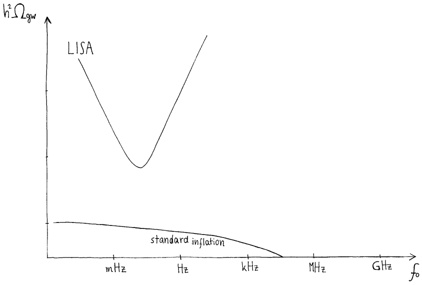
figure from www.esa.int/ESA Multimedia/If our eyes could see gravitational waves

vacuum tensor perturbations during inflation

$$\delta T^t_{ij}{=}0\ ,\quad h^t_{ij}{=}\textstyle\sum_\lambda \epsilon^{(\lambda)}_{ij}\, h^{(\lambda)} \quad \Rightarrow \quad \left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right) h^{(\lambda)}(t,k) = 0$$

$$\curvearrowright \quad \langle 0|h^{(\lambda)}(t,\mathbf{x})\, h^{(\lambda)}(t,0)|0\rangle_{w_\mathbf{k}} = \int \mathrm{d}\ln k\ \mathcal{P}_h\ , \qquad \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} = \frac{k^3\,\mathrm{d}\ln k}{2\pi^2}$$

$$\mathcal{P}_{\mathrm{T}}(k) = \underbrace{32\pi G}_{\text{canonical normalization}} \times \underbrace{2}_{\sum_\lambda} \times \mathcal{P}_h = \frac{16}{\pi}\left(\frac{H}{m_{\mathrm{pl}}}\right)^2 \qquad \text{flat spectrum}$$

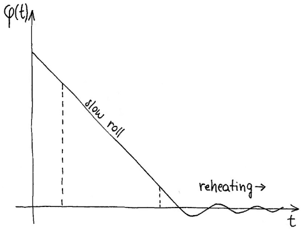


shortcomings of inflation

* **strong constraints** on the form of the **potential**
* what are the **initial conditions?**
* **transition to radiation-dominated epoch?**

a thermal treatment of inflation

* scalar field $\varphi = \varphi(t)$
* self-interaction potential $V(\varphi)$
* medium at T
* friction Υ transfers energy from φ to medium
  $\Rightarrow$ many time scales
* how is φ coupled to the heat bath?



equations of motion

$$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - V(\varphi) - \varphi J + \mathcal{L}_{\texttt{bath}}$$

$$\downarrow$$

$$\ddot{\varphi} + 3H\dot{\varphi} + V_\varphi(\varphi) + \langle J(t)\rangle = 0\,, \qquad V_x \equiv \partial_x V$$

$$\downarrow$$

$$\langle J(t)\rangle = -\int_0^t dt'\varphi(t')\,\underbrace{C_{\text{R}}(t-t')}_{\text{retarded correlator}} + \mathcal{O}(J^3)$$

$$\downarrow$$

$$\boxed{\ddot{\varphi} + (3H + \Upsilon)\dot{\varphi} + V_\varphi(\varphi, m_{\text{T}}) \approx 0}$$

$$\Upsilon \approx \frac{\mathrm{Im}\,C_{\text{R}}(m)}{m}\,, \qquad m_{\text{T}}^2 \approx m^2 - \mathrm{Re}\,C_{\text{R}}(m)$$

benchmark solutions

$$\mathcal{L} = \frac{1}{2}\partial^\mu\varphi\,\partial_\mu\varphi - V(\varphi) - \varphi J + \mathcal{L}_{\texttt{bath}}$$

impose symmetry $\Rightarrow$ φ pseudoscalar

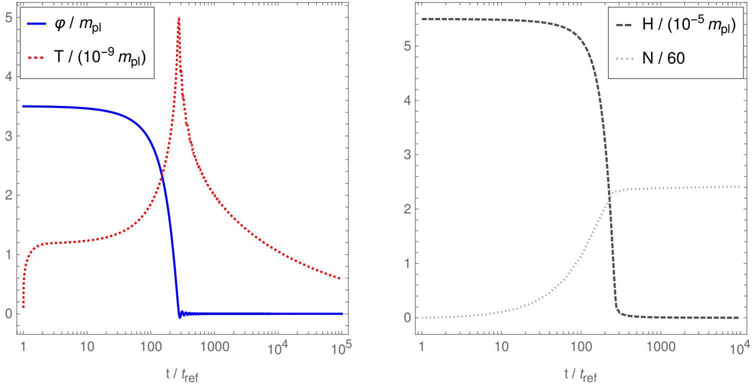
* axion-like coupling: $J = \frac{g^2}{f_a}\frac{\epsilon^{\mu\nu\rho\sigma}F^c_{\mu\nu}F^c_{\rho\sigma}}{64\pi^2}$

  f_a decay const. , $\alpha = \frac{g^2}{4\pi}$ YM coupling, $c \in \{1, \ldots, N_c^2 - 1\}$

* periodic potential: $V(\varphi) \simeq m^2 f_a^2 [1 - \cos(\frac{\varphi}{f_a})]$

non-Abelian gauge fields $\Rightarrow$ medium thermalizes fast

benchmark solutions



$f_a \sim m_{\mathrm{pl}}$, $m/m_{\mathrm{pl}} \sim 10^{-6}$, $\Lambda_{\mathrm{IR}}/\mathrm{GeV} \sim 0.2$, $t_{\mathrm{ref}} \sim H(0)^{-1}$

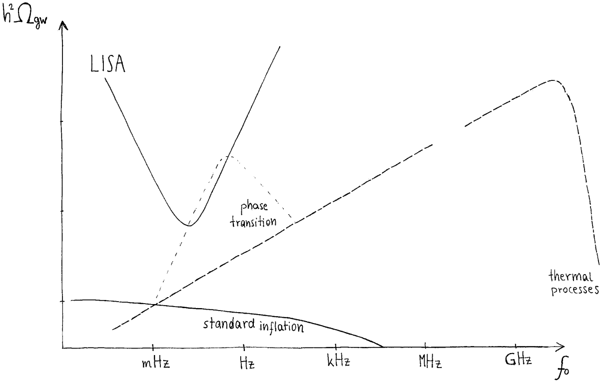
gravitational waves from thermal processes

$$\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right) h^{(\lambda)}(t,\mathbf{k}) = 16\pi G\, \epsilon_{ij}^{(\lambda)}\, T_{ij}^{t}(t,\mathbf{k}) \equiv \rho^{(\lambda)}(t,\mathbf{k})$$

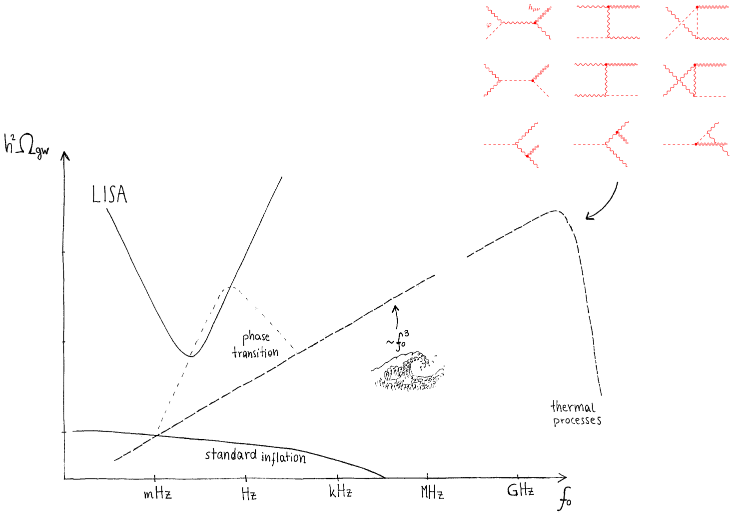
$$\Rightarrow \quad h^{(\lambda)}(t,\mathbf{k}) = \int_{-\infty}^{t} \mathrm{d}t_i \, \underbrace{G_{\mathrm{R}}(t,t_i,k)}_{\text{Green's function}} \, \rho^{(\lambda)}(t_i,\mathbf{k})$$

$$\curvearrowright \quad \langle h(t,\mathbf{x})\, h(t,0)\rangle_{\text{vacuum + thermal}} = \ldots$$

gravitational waves from thermal processes



gravitational waves from thermal processes



hydrodynamic regime

$$T^{\mu\nu}_{\texttt{hydro}} = \underbrace{\bar{T}^{\mu\nu}}_{(\bar{T},0)} + \underbrace{\delta T^{\mu\nu}_{\texttt{hydro}}}_{(\delta T, v^i)} + \underbrace{S^{\mu\nu}}_{\text{thermal noise}} + \mathcal{O}(\delta^2)$$

- $S^{\mu\nu}$: fluctuation-dissipation relation
- polarization sum $\sum_\lambda \epsilon^{(\lambda)}_{ij} \epsilon^{(\lambda)*}_{mn} = \mathbb{L}_{ij;mn}$
- $\langle S^{ij} S^{mn} \rangle \sim T \left[\underbrace{\eta}_{\text{shear}} (\delta_{im}\delta_{jn} + \delta_{in}\delta_{jm}) + (\underbrace{\zeta}_{\text{bulk}} - \frac{2\eta}{3}) \delta_{ij}\delta_{mn} \right]$

$$\mathbb{L}_{ij;mn} \langle T^{ij}_{\texttt{hydro}} \, T^{mn}_{\texttt{hydro}} \rangle \sim \frac{T\eta}{a^4} \sim \begin{cases} \Upsilon^{-1} & m \ll T \\ \left(\frac{T}{g}\right)^4 & m \gg T \end{cases}$$

vacuum + thermal fluctuations

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{16}{\pi}\left(\frac{H}{m_{\mathrm{pl}}}\right)^2 + \frac{(32)^2 k^3}{m_{\mathrm{pl}}^4}\int_{-\infty}^{t_e} \mathrm{d}t_i\, \underbrace{G_{\mathrm{R}}^2(t_e, t_i, k)}\, \overbrace{T(t_i)\,\eta(t_i)}^{\text{model dependence}}$$

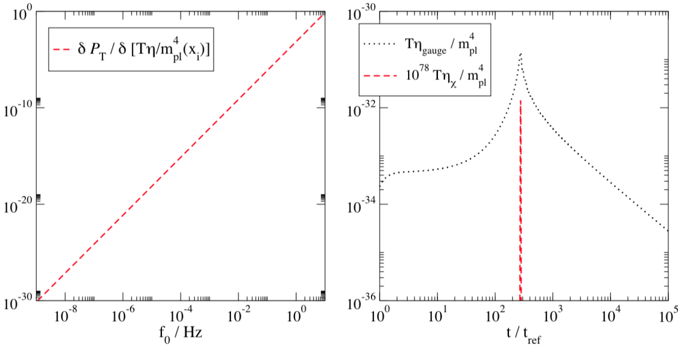
$$\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right) G_{\mathrm{R}} \;=\; 0$$

$$k \ll aH \quad \Rightarrow \quad \left(\partial_t^2 + 3H\partial_t\right) G_{\mathrm{R}} \;\approx\; 0$$

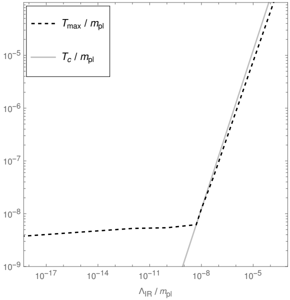
independent of $k = 2\pi \underbrace{a_0 f_0}_{\text{today}}$ $\Rightarrow$ f_0^3 shape is model-independent!

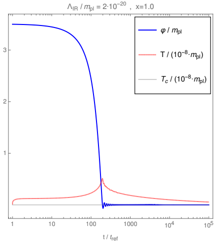
vacuum + thermal fluctuations

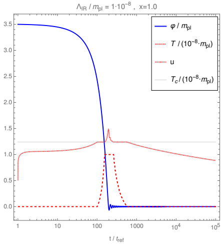
$$\mathcal{P}_{\mathrm{T}}(k) = \frac{16}{\pi}\left(\frac{H}{m_{\mathrm{pl}}}\right)^2 + \underbrace{\frac{(32)^2 k^3}{m_{\mathrm{pl}}^4}\int_{-\infty}^{t_e} \mathrm{d}t_i\, G_{\mathrm{R}}^2(t_e, t_i, k)\, T(t_i)\,\eta(t_i)}$$

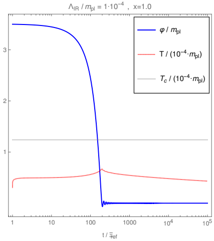


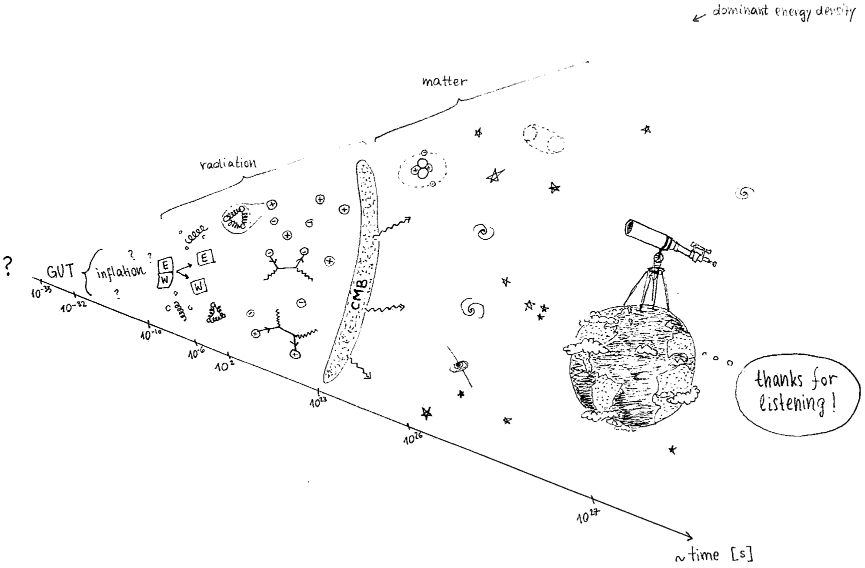
$T_{\max}$ reached after inflation depends on the physics of the plasma











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thanks for listening!