hadronic contributions to the muon $g-2$ from lattice QCD

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12 June 2023

AEC Plenary Meeting 2023

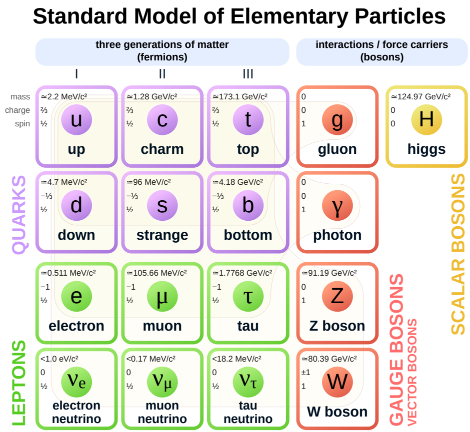
the muon and its magnetic dipole moment

the muon [Anderson, Neddermeyer 1936]

- an elementary particle
- an heavier electron, $m_\mu \approx 105.66\,\mathrm{MeV} \approx 207 \times m_e$
  "Who ordered that?" — I. I. Rabi
- spin $1/2 \Rightarrow \vec{S} = \hbar\vec{\sigma}/2$

a spinning charge in a magnetic field $\vec{B}$ couples to it:
$\vec{\mu}\cdot\vec{B}$, where the dipole moment is

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$$



- precession of the magnetic moment
- the Landé g-factor for orbital angular momentum $\vec{L}$ is $g_L = 1$
- Dirac's relativistic wave equation with minimal coupling
  $\Rightarrow g = 2$ for spin angular momentum [Dirac 1928]

the anomalous magnetic moment

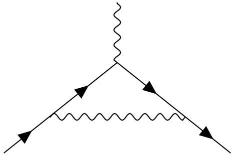
Dirac's theory of charged spin-$1/2$ particles: $g = 2$ [Dirac 1928]

quantum effects in the full interactive quantum field theory modifies this prediction

$\Rightarrow$ the anomalous magnetic moment

$$a = \frac{g-2}{2} = F_2(0)$$

$$\langle \ell(p') | j_\mu(0) | \ell(p) \rangle = \bar{\ell}(p') \left[\gamma_\mu F_1(q^2) + \frac{\mathrm{i}}{2m_\ell} \sigma_{\mu\nu} q^\nu F_2(q^2) + \dots \right] \ell(p), \qquad q = p' - p$$



- at $\mathcal{O}(\alpha)$, $a_\ell = \frac{\alpha}{2\pi} \approx 0.001\,161\,41$ for every lepton, one of the first calculations in QED [Schwinger 1948]
- first experimental measurement, $a_e = 0.001\,19(5)$ [Kursch, Foley 1948]

why the muon?

SM precision physics (a_ℓ) is sensitive to loops of new BSM degrees of freedom

- CP and flavour conserving, chirality flipping process: for a particle of mass $M \gg m_\ell$

$$a_\ell^{\text{new phys.}} \sim \left(\frac{\Delta_{LR}}{m_\ell}\right)\left(\frac{m_\ell}{M}\right)^2$$

- for Yukawa and weak interactions, $\Delta_{LR} = m_\ell$

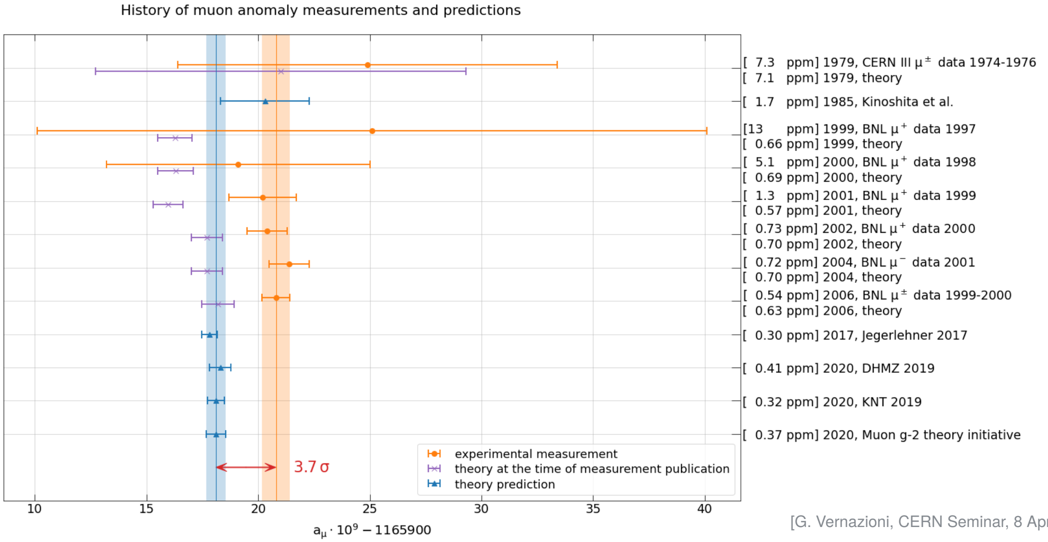
$\Rightarrow$ measure and compute a_ℓ precisely to test for new physics!

among elementary leptons, the muon is the best suited

$$m_e \approx 0.511\,\text{MeV}, \qquad m_\mu \approx 106\,\text{MeV}, \qquad m_\tau = 1\,776.86(12)\,\text{MeV}$$

- a_μ is $(m_\mu/m_e)^2 \approx 40\,000$ times more sensitive to new physics than a_e
- a_τ is even more sensitive, but τ half-life is too short
  $\Rightarrow$ experimentally $-0.052 < a_\tau < 0.013$

the anomalous magnetic moment – theory vs. experiment



[G. Vernazioni, CERN Seminar, 8 Apr. 2021]

the anomalous magnetic moment of the muon, today

status from the Muon $g-2$ Theory initiative white paper [Aoyama *et al.*, MC Physics Reports 887 (2020)]

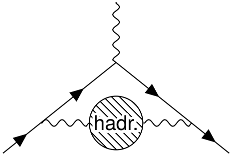
and the recent Fermilab Muon $g-2$ experiment result [Abi *et al.*, Phys. Rev. Letter **126**, 141801 (2021)]

	$a_\mu \times 10^{11}$		
QED	$116\,584\,718.931(104)$	up to 10th order	
EW	$153.6(1.0)$	two loops	
HVP, LO	$6\,931(40)$	e^+e^-	
	$7\,116(184)$	lattice, $udsc$	
HVP, NLO	$-98.3(7)$	e^+e^-	
HVP, NNLO	$12.4(1)$	e^+e^-	
HLbL	$90(17)$	pheno. + lattice	
total SM	$116\,591\,810(43)$	Muon $g-2$ Theory Initiative	[Aoyama *et al.*, MC 2020]
experiment	$116\,592\,089(63)$	BNL E821	[Bennett *et al.* 2006]
	$116\,592\,040(54)$	FNAL E989 (Muon $g-2$ exp.)	[Abi *et al.* 2021]
	$116\,592\,061(41)$	experimental average	

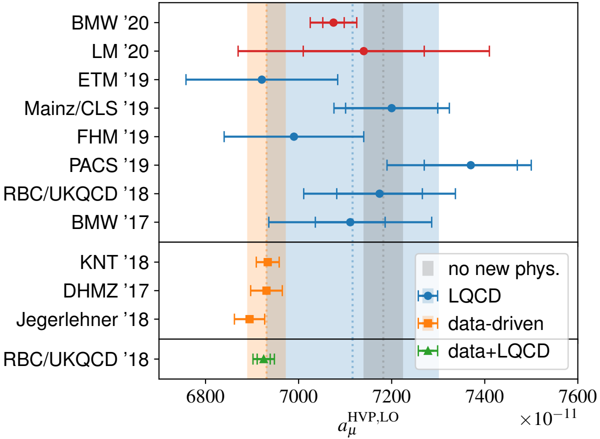
$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 251(59) \times 10^{-11}$$, a 4.2σ tension

- initial FNAL result based on $6\,\%$ of target statistics $\Rightarrow$ $\delta a_\mu \approx 15 \times 10^{-11}$ with full statistics
- upcoming J-PARC E34 (Muon $g-2$/EDM experiment) with novel approach [Abe *et al.* 2019]

the leading-order HVP contribution to a_μ



- extracted from the exp. R-ratio data via dispersive integral (data-driven method)
- or computed on the lattice with a known QED kernel
- or measured from $\Delta\alpha_{\rm had}(t)$ with $t < 0$ in t-channel scattering experiments



- no new physics: the value of $a_\mu^{\rm HVP,LO}$ that matches the experimental result without BSM contributions
  $\Rightarrow$ 4.2σ tension with the data-driven estimate
- BMW '20 result: 2.1σ tension with the pheno. estimate, 1.5σ from "no new physics"
- Fermilab target $\delta a_\mu \approx 15 \times 10^{-11}$, $2\,‰$ of $a_\mu^{\rm HVP,LO}$ $\Rightarrow$ we want a per-mille level theory prediction

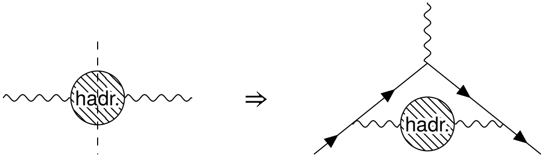
the leading-order HVP contribution – data-driven estimate

data driven, based on the experimental data for the hadronic cross-section ratio R as a function of s

$$R(s) = \frac{\sigma_{e^+e^- \to \text{hadrons}}(s)}{\sigma_{e^+e^- \to \mu^+\mu^-}(s)}$$

collected over the years by many experiments, channel by channel below $2\,\text{GeV}$ and narrow resonances

- scan method: BESII, BESIII at IHEP, CMD-2, CMD-3, SND at BINP, many older experiments …
- initial-state radiation (ISR): KLOE at DAΦNE, KEK-B, BaBar at PEP-II, BESIII, CLEO-c, …



$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \mathrm{d}s\, \frac{\hat{K}(s)}{s^2} R(s), \qquad \hat{K}(4m_\pi^2) \approx 0.63, \quad \lim_{s\to\infty} \hat{K}(s) = 1$$

- $\hat{K}(s)$ is a known QED kernel
- interpolation, averaging, integration of the $R(s)$ data
  - Davier, Hoecker, Malaescu, Zhang, …(DHMZ) • Keshavarzi, Nomura, Teubner, …(KNT) • Jegerlehner

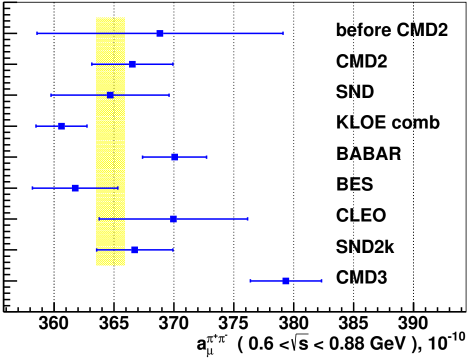
the leading-order HVP contribution – the $\pi^+\pi^-$ channel

of the hadronic cross-section represents $70\,\%$ of the total contribution to (and of the error of) $a_\mu^{\mathrm{HVP,LO}}$

long-standing tension between KLOE and BABAR

⇒ recent result by CMD-3 detector at VEPP-2000
is precise and significantly larger [arXiv:2302.08834]

- not obvious how to combine these different results in a single data-driven estimate of $a_\mu^{\mathrm{HVP,LO}}$
- potential solution of the $g-2$ puzzle?



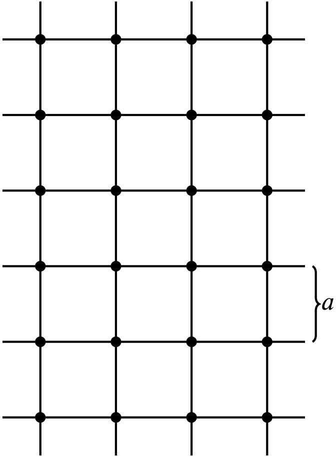
⇒ additional motivation for a first-principle calculation of $a_\mu^{\mathrm{HVP,LO}}$ on the lattice

quantum gauge theories on the lattice

four-dimensional lattice with spacing $a \Rightarrow$ regularization of the theory [Wilson 1974]

- gauge bosons (gluons) live on links between sites
- fermions (quarks) live on sites and are integrated out

$$\langle j_5(x) j_5(0) \rangle = \int \mathrm{d}U_\mu(x) \left| D[U]^{-1}(x,0) \right|^2 \exp\left\{ -S_g[U] \right\} \det D[U]^2$$



finite box $L^3 \times T \Rightarrow 8 \times 4 \times L^3 \times T$ = millions of integrals

$\Rightarrow$ the lattice theory is solved numerically on supercomputers,
using Monte Carlo integration with importance sampling

- Euclidean space-time is needed for a real, positive-definite weight
- generate an ensemble $\{U_i\}$ of gauge field configurations with Boltzmann weight

$$\exp\left\{ -S_g[U] \right\} \det D[U]^2 = \exp\left\{ -S_g[U] - \left| D[U]^{-1} \phi \right|^2 \right\}$$

- sample the observable $\left| D[U]^{-1}(x,0) \right|^2$ on the ensemble

the leading-order HVP contribution – on the lattice

using the time-momentum representation (TMR) method [Bernecker, Meyer 2011; Francis *et al.* 2013]

$$a_\mu^{\mathrm{HVP,LO}} = \int_0^\infty \mathrm{d}t\, w(t) G(t),$$

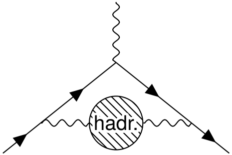
with a known kernel function

$$w(t) = 4\pi^2 \int_0^\infty \mathrm{d}Q^2\, f(Q^2) K(t, Q^2), \qquad K(t, Q^2) = t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)$$

and the zero-momentum-projected correlator of Euclidean-time t of the electromagnetic current $j_\mu(x)$

$$G(t) = -\frac{1}{3}\int \mathrm{d}^3x \sum_{k=1}^{3} \langle j_k(x) j_k(0) \rangle$$

the leading-order HVP contribution to a_μ



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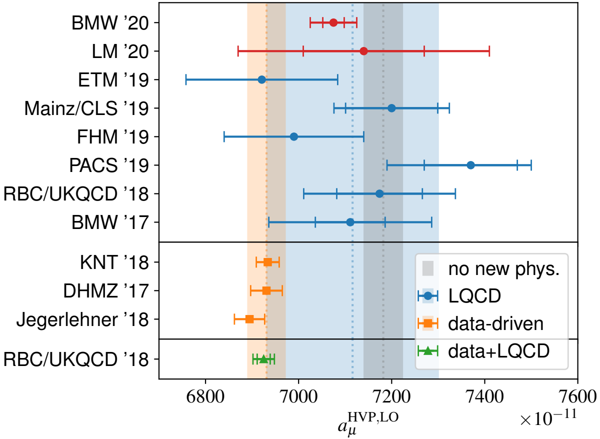

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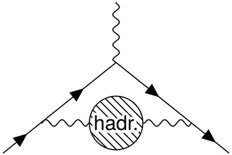

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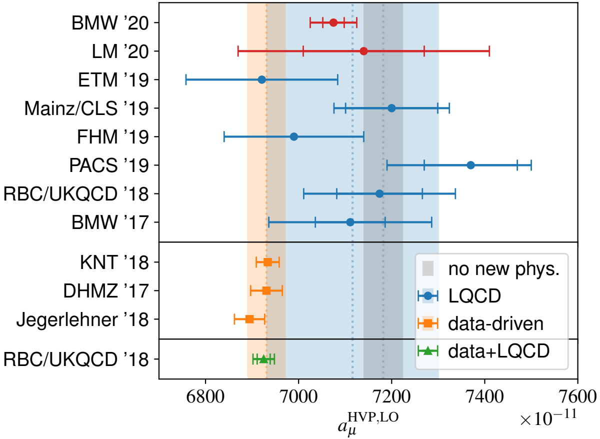

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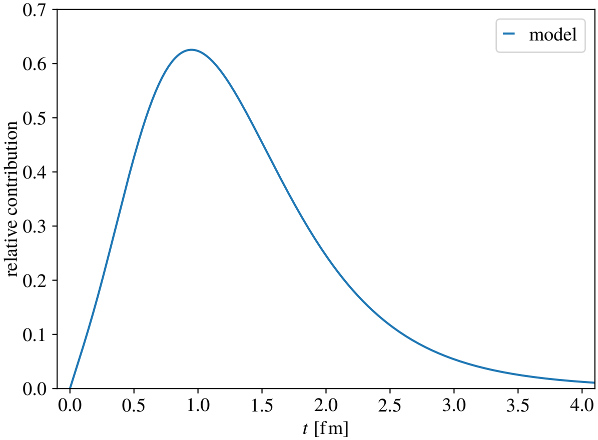
the leading-order HVP contribution – the TMR method

$$a_\mu^{\mathrm{HVP,LO}} = \int_0^\infty \mathrm{d}t\, w(t) G(t),$$



using a model for the Euclidean-time correlator

[Bernecker, Meyer 2011]

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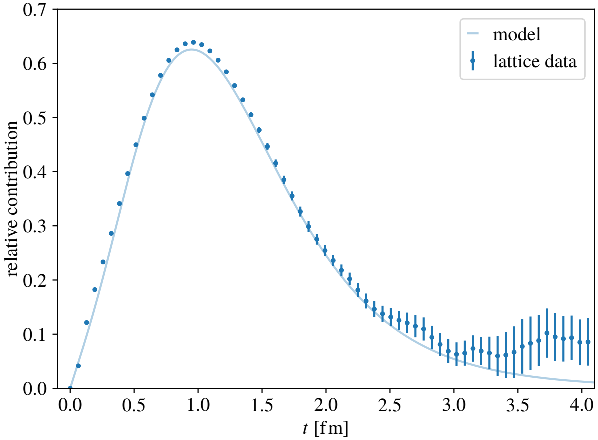

using lattice data at physical m_π for the Euclidean-time correlator (connected contribution only)

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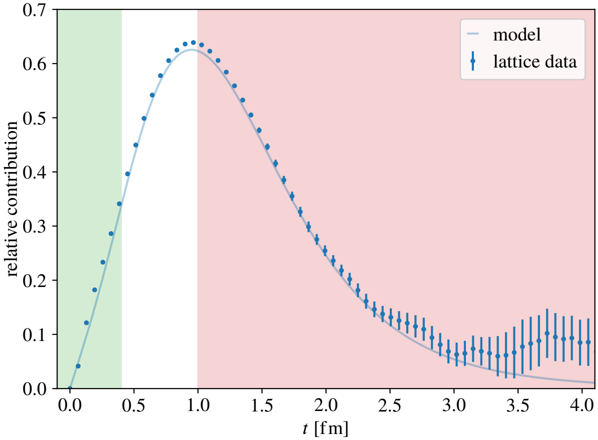


using lattice data at physical m_π for the Euclidean-time correlator (connected contribution only)

systematic effects to the per-mille level

controlling the tail of the correlator at large t ⇒ main source of statistical uncertainty

- especially for the disconnected ⇒ one-end trick very effective [McNeile, Micheal 2006; Giusti, Harris, Nada, Schaefer 2019]
- use the bounding method [Lehner LGT2016; Gérardin, MC *et al.* 2019]

extrapolation to the continuum limit (and extra- or interpolation to physical meson masses)

- ensembles around physical meson masses are used by almost all collaborations
- higher order terms ($\sim a^3$, a^4) are important especially at small t
- possible scaling violations ($\sim \log a$) at small t? [MC *et al.* 2021, Husung *et al.* 2022, Sommer Lattice 2022]

simulations of finite-size lattices ⇒ correction of finite-size effects

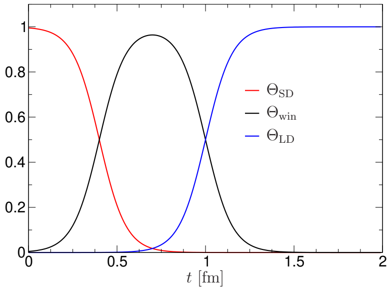
- computing FSE on the zero-momentum correlator $G(t)$ [Lüscher 1991; Lellouch, Lüscher 2000; Meyer 2011]
- significant progress in the last few year, mostly solved [Hansen, Patella 2019; 2020]

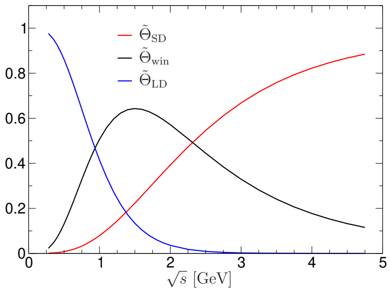
QED and strong isospin breaking corrections, scale setting systematics

- small (percent level) contribution, but essential to obtain a per-mille result
- a $1\,\%$ scale uncertainty is a $\approx 2\,\%$ systematic error on $a_\mu^{\mathrm{HVP,LO}}$ ⇒ a per-mille level scale determination is needed
- agree on a common scheme! [“Converging on QCD+QED prescriptions” workshop in Edinburgh, 29-31 May 2023]

the leading-order HVP contribution – Euclidean-time windows

$$a_\mu^{\mathrm{HVP,W}} = \int_0^\infty \mathrm{d}t\, w(t) G(t) \left[\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)\right], \qquad \Theta(t, t', \Delta) = \frac{1}{2}\left(1 + \tanh\frac{t - t'}{\Delta}\right)$$

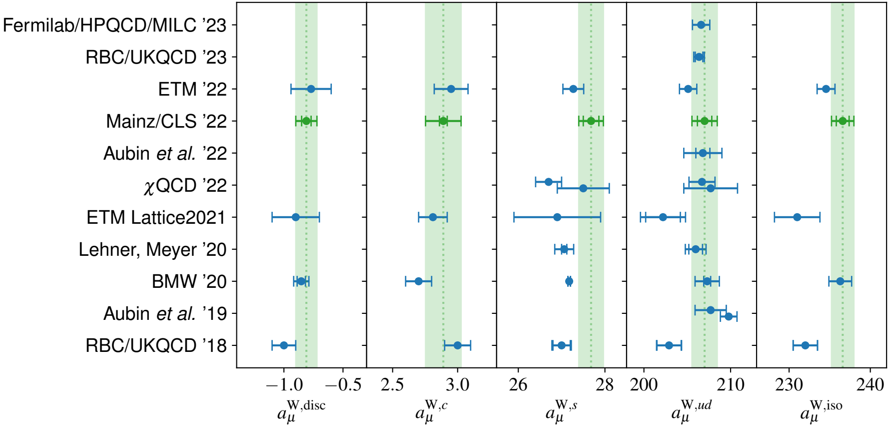




- $t_0 = 0.4\,\mathrm{fm}$, $t_1 = 1.0\,\mathrm{fm}$, $\Delta = 0.15\,\mathrm{fm}$
- originally introduced to combine lattice QCD and data-driven [Blum *et al.* (RBC/UKQCD) 2018]
- powerful tool to compare lattice QCD results from different collaboration in well-defined exclusive regions
- (smoothed) compact support in Euclidean time $t \Rightarrow$ broad region in $\sqrt{s}$ [figures from Colangelo *et al.* 2022]

  $\Rightarrow$ comparison with the data-driven method is still possible

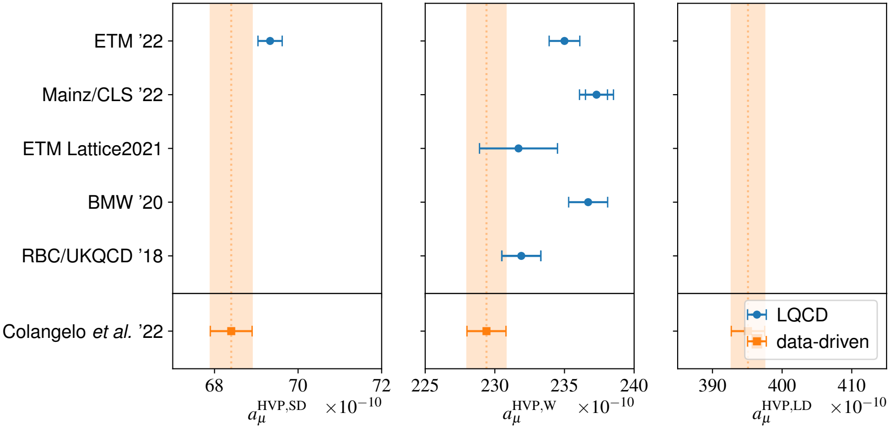
the leading-order HVP contribution – intermediate window results



- lattice computations are most precise in the (intermediate) window ⇒ discussion on systematics
- the latest result by Mainz/CLS is highlighted in green [MC *et al.* (Mainz/CLS) 2022]
- confirmed by independent lattice results from different collaborations

[Alexandrou *et al.* (ETM) 2022; Blum *et al.* (RBC/UKQCD) 2023; Bazavov *et al.* (Fermilab/HPQCD/MILC) 2023]

the leading-order HVP contribution – lattice and data-driven windows



W: the intermediate window computed on the lattice is in tension with the data-driven estimate!

SD: also hints of a tension [Alexandrou *et al.* (ETM) 2022]

RBC/UKQCD 2023: light connected only, agrees with ETM 2022 [Blum *et al.* (RBC/UKQCD) 2023]

LD: no result yet for the long-distance window (and the full $a_\mu^{\mathrm{HVP,LO}}$)

the leading-order HVP contribution – conclusions and outlook

W: several lattice calculations confirm BMW ’20 result on the (intermediate) window

- including a blind analysis [*e.g.* Blum *et al.* (RBC/UKQCD) 2023]
  $\Rightarrow$ more collaboration will employ blinding in the future
- the discrepancy between lattice and data-driven is a serious puzzle
  in principle, this is not limited to the muon $g-2$

SD: only one complete result so far [Alexandrou *et al.* (ETM) 2022]

- light connected only from RBC/UKQCD 2023, agrees with ETMc 2022 [Blum *et al.* (RBC/UKQCD) 2023]
- work in progress, several results expected soon!
- the kernel $\sim t^4$ for $t \to 0 \Rightarrow \log a$ cut-off effects [MC, Harris, Meyer, Toniato, Török 2021]
  $\Rightarrow$ modifying the kernel approach at $t \to 0$? [Sommer Lattice 2022]

LD: the long-distance window ($\Rightarrow$ full $a_\mu^{\mathrm{HVP,LO}}$) requires significant computational investment

- improved bounding method $\Rightarrow$ significant computational investment, high priority
- alternative ideas to address the S/N problem? [Dalla Brida, Giusti, Harris, Pepe 2020]

- SD + W + LD $\Rightarrow$ full contribution
- other proposals for windows with different weights in $\sqrt{s}$ [e.g. Boito *et al.* 2022]
  - reconstructing $R(s)$ from Euclidean-time is ill posed $\Rightarrow$ regularization/smearing [Alexandrou *et al.* (ETM) 2023]
  - the HVP function $\bar{\Pi}(-Q^2)$ at space-like $Q^2 > 0 \Rightarrow$ connection with the running of α [MC *et al.* 2022]

thanks

for your attention!

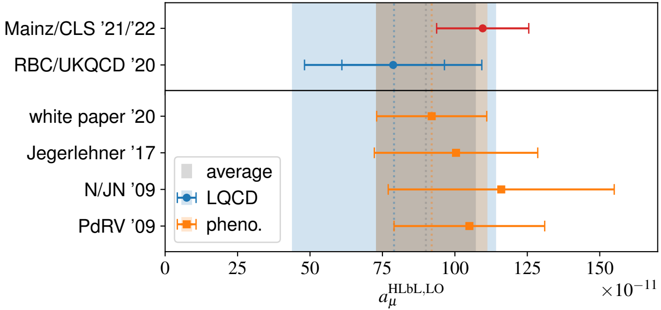
questions?

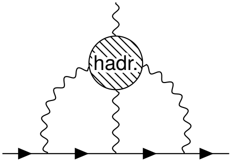
backup slides

the leading-order HLbL contribution to a_μ

until a few years ago, “Glasgow consensus” $a_\mu^{\text{HLbL,LO}} = 105(26) \times 10^{-11}$ [Prades, de Rafael, Vainshtein 2009]

hadronic model + perturbative QCD ⇒ large systematic uncertainty





- theory white paper data-driven dispersive $a_\mu^{\text{HLbL,LO}} = 92(19) \times 10^{-11}$
- RBC/UKQCD lattice result $a_\mu^{\text{HLbL,LO}} = 79(35) \times 10^{-11}$ [Blum *et al.* 2020]
- combined theory white paper: $a_\mu^{\text{HLbL,LO}} = 90(17) \times 10^{-11}$
- Mainz lattice result $a_\mu^{\text{HLbL,LO}} = 109.6(15.9) \times 10^{-11}$ (not included in the theory white paper) [Chao *et al.* 2021; 2022]

⇒ the error is on much more solid ground

⇒ highly unlikely that the HLbL contribution can have a rôle in solving the muon $g-2$ puzzle