Merons:
How to identify
the carriers of topological charge

Stephan Caspar

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Albert Einstein Center for Fundamental Physics
Institute for Theoretical Physics
University of Bern

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Manes Hornung, João Pinto Barros,
Uwe-Jens Wiese (AEC, Bern)
and Wolfgang Bietenholz (UNAM, Mexico)

Overview

O(N)-Models
- O(N) Quantum Field Theory
- Topology

Merons
- Wolff Clusters
- Clusters and Topology

Results
- 1d-O(2)
- 2d-O(3)
- 3d-O(4)

O(N)-Models

O(N): Prototypical Topological Field Theory

Non-linear σ-models with $O(N)$ rotational symmetry

$$S = \frac{1}{2g^2} \int d^d x \, (\partial_\mu \vec{e})^2$$

- Interacting theory through non-linear constraint $|\vec{e}|^2 = 1 \Leftrightarrow \vec{e} \in S^{N-1}$
- For $d = N - 1$ a topological charge can be defined

$$Q = \frac{1}{V(S^{N-1})} \int d^{N-1}x \, \epsilon_{\mu_1 \ldots \mu_{N-1}} \epsilon^{i_1 \ldots i_N} \, e^{i_N} \prod_{j=1}^{N-1} \partial_{\mu_j} e^{i_j}$$

(Counting how many times the field winds around S^{N-1})

O(N): Topological Sectors

- $Q \in \mathbb{Z}$ is always an integer
- Invariant under local changes of the field configuration
- Phase-space (and partition function) separable by charge

$$Z = \sum_{Q} Z_Q$$

- Defines a family of different θ-vacuua theories

$$S \to S - i\theta Q, \qquad Z(\theta) = \sum_{Q} Z_Q \exp(i\theta Q) \quad (0 \leq \theta < 2\pi)$$

Merons

O(N): Wolff Clusters

- Lattice regularization $\vec{e}_x,\ x \in X \subset \mathbb{R}^{N-1}$
- Sampling partition function using non-local updates: Clusters $\mathcal{C} \subset X$ of spins are reflected collectively and independently along some direction $\vec{n} \in S^{N-1}$
- Boltzmann weight is distributed among all possible cluster breakups $\{\mathcal{C}\}$

$$\exp(-S(\{\vec{e}\})) = \sum_{\{\mathcal{C}\}} W(\{\vec{e}\},\{\mathcal{C}\})$$

(Wolff, 1989)

Topological Charge of Clusters

- Charge Q can be obtained from piecewise differentiable interpolated field
- Lattice field $\vec{e}_x$ continously trivializable to $\vec{e}_x = \text{const.}$
- Solution: Zero weight for exceptional configurations (where Q is discontinous)

The Meron

Action barrier can be chosen such that the topological charge factorizes

$$Q = \sum_{\mathcal{C}} Q_{\mathcal{C}}$$

Cluster topological charge changes sign under reflection of the cluster $Q_{\mathcal{C}} \to -Q_{\mathcal{C}}$:

- Clusters carry half integer charge $Q_{\mathcal{C}} = \Delta Q/2$
- Clusters with $Q_{\mathcal{C}} = \pm\frac{1}{2}$ are called (anti-)merons (Bietenholz et al., 1995)

Results

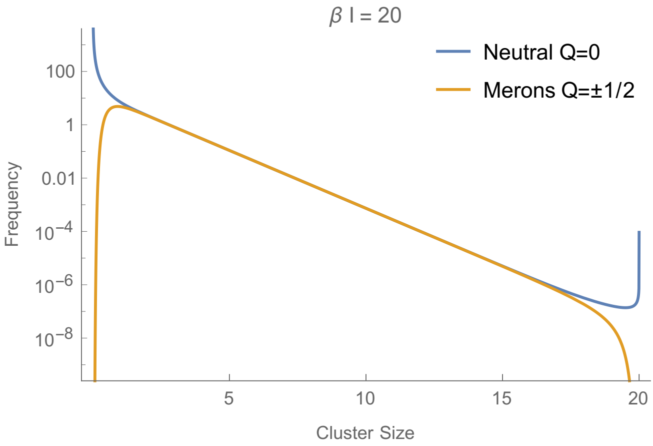
1d-O(2): A quantum mechanical example

- Corresponds to a particle on a circle S^1
- Spectrum and transfer matrix known analytically

$$E_k = \frac{I}{2}k^2, \quad k \in \mathbb{Z}$$

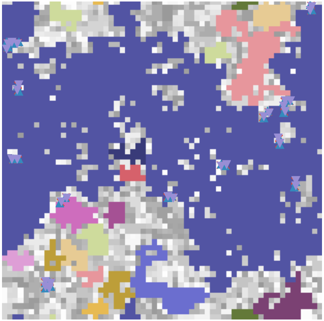
- Cluster decomposition analytically calculable
- Restricted cluster charges $Q = 0, \pm 1/2$

1d-O(2): Cluster Size Distribution

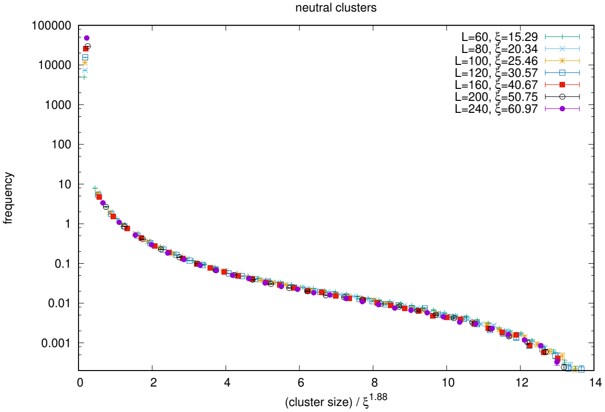


2d-O(3): Asymptotic Free Field Theory

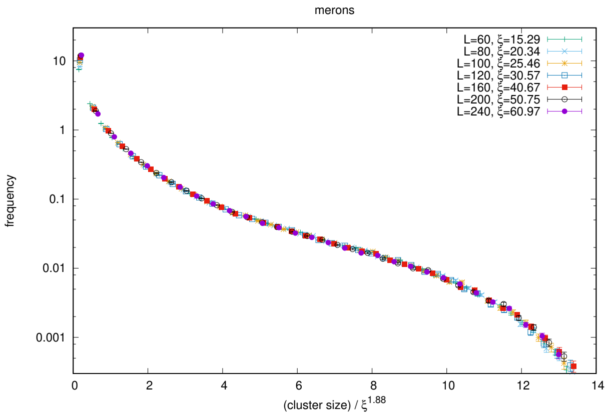
- Clusters have fractal-like structures at all scales
- Cluster-size becomes cutoff dependent
- Running coupling = no self-similarity
- Clusters appear to have a fractal dimension $D \approx 1.88$



2d-O(3): Cluster Size Distribution



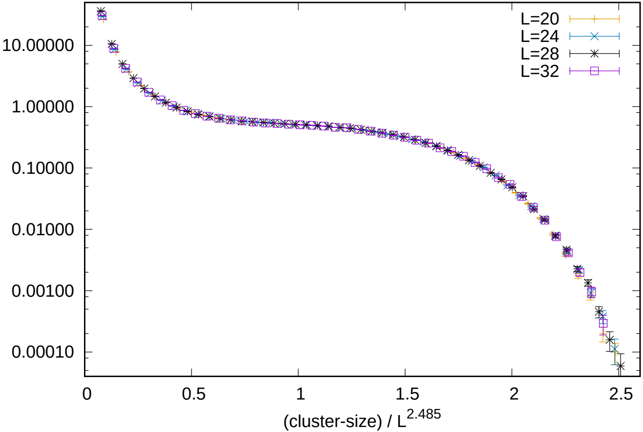
2d-O(3): Cluster Size Distribution



3d-O(4): 2nd order fix-point

- Continuum Limit at constant coupling
- Natural self-similarity
- Clusters fractal dimension $D \approx 2.485$

3d-O(4): Cluster Size Distribution



3d-O(4): Status

- Topological sectors not yet separated
- Fix-point may not be accessible with infinte action barriers

Outlook

2d-O(3):

- Determining distributions for higher charges $Q = 1, \ldots$
- Requires mores statistics, smaller cutoff
- Determining the origin of the divergent topological suszeptibility $\chi_t = \langle Q^2 \rangle \to \infty$
- Theoretical explanation for pseudo-fractality

3d-O(4):

- Investigate separation of topological sectors

Thank you for your attention